

# Exercises 3:15-4:00

Three simplified case studies.

Purpose: test your modelling capabilities

# Case study 1: LMCFP

Linear single-commodity Minimum-Cost Problem.

If time permits, else read alone.

Yearly demands amount to 90 000, 80 000, 50 000 and 70 000 items, respectively. The transport costs per item from the assembly plant of Rotterdam to the warehouses of Bristol and Middlesbrough are € 24.5 and € 26.0, respectively, whereas the transport costs per item from the warehouses to the sales districts are reported in Table 6.2. Both warehouses have an estimated capacity of 15 000 items and are supplied 10 times a year. Consequently their maximum yearly throughput is 150 000 items.

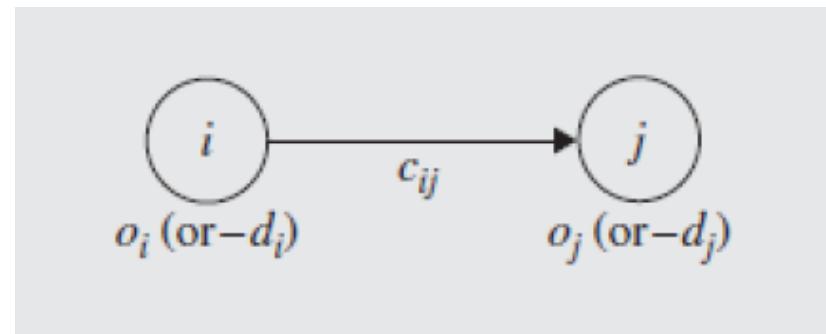
Table 6.2 Transport costs (in €) per item from the warehouses to the sales districts in the Boscheim problem.

Warehouse	Sales districts			
	London	Birmingham	Leeds	Edinburgh
Bristol	9.6	7.0	15.2	28.5
Middlesbrough	19.5	13.3	5.0	11.3

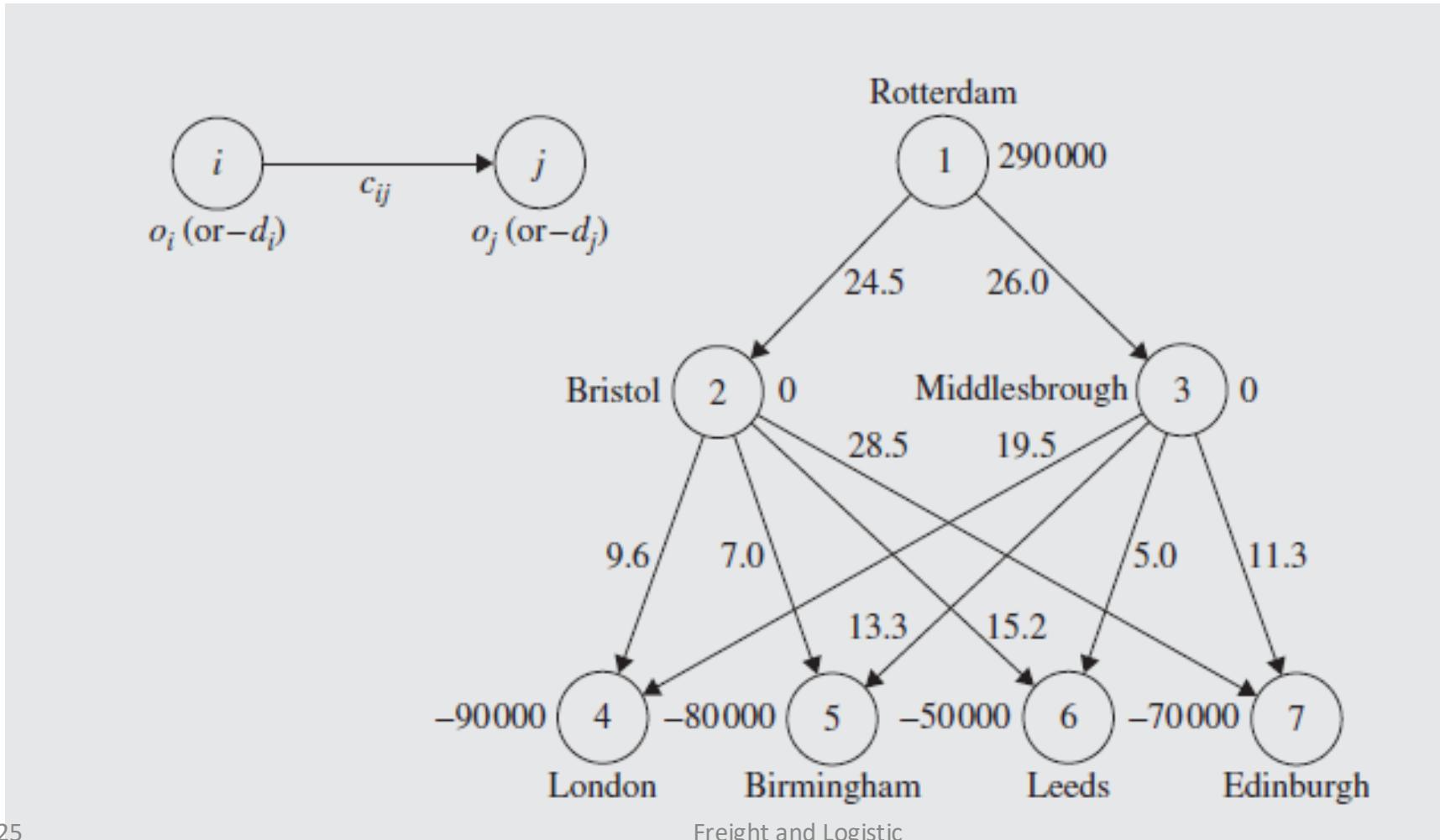
# Questions

- (a) Provide a graph representation
- (b) Write the linear programming formulation. We just ask for the formulation (objective function and constraints)

# Hints on notations



# (a) Graph representation



## (b) Linear single-commodity minimum-Cost Problem

$$\begin{aligned} \text{Minimize } & 24.5x_{12} + 26.0x_{13} + 9.6x_{24} + 7.0x_{25} + 15.2x_{26} + 28.5x_{27} \\ & + 19.5x_{34} + 13.3x_{35} + 5.0x_{36} + 11.3x_{37} \end{aligned}$$

← 10 transportation costs

subject to

$$x_{12} + x_{13} = 290\,000 \quad \leftarrow \text{Rotterdam}$$

$$x_{24} + x_{25} + x_{26} + x_{27} - x_{12} = 0 \quad \leftarrow \text{Inflow} = \text{Outflow}$$

$$x_{34} + x_{35} + x_{36} + x_{37} - x_{13} = 0$$

$$-x_{24} - x_{34} = -90\,000$$

$$-x_{25} - x_{35} = -80\,000$$

$$-x_{26} - x_{36} = -50\,000 \quad \leftarrow \text{Final destinations}$$

$$-x_{27} - x_{37} = -70\,000$$

$$x_{12} \leq 150\,000$$

← Constraints on warehouses

$$x_{13} \leq 150\,000$$

$$x_{12}, x_{13}, x_{24}, x_{25}, x_{26}, x_{27}, x_{34}, x_{35}, x_{36}, x_{37} \geq 0.$$

← Non-negativity constraints

# Solution: Total transportation cost is: €2,591,000

Route	Warehouse	Sales District	Items Shipped
From Bristol to London	Bristol	90,000	90,000
From Bristol to Birmingham	Bristol	60,000	60,000
From Bristol to Leeds	Bristol	0	0
From Bristol to Edinburgh	Bristol	0	0
From Middlesbrough to London	Middlesbrough	0	0
From Middlesbrough to Birmingham	Middlesbrough	20,000	20,000
From Middlesbrough to Leeds	Middlesbrough	50,000	50,000
From Middlesbrough to Edinburgh	Middlesbrough	70,000	70,000

# Case study 2: maintenance

Discussion in class

# Open question

- (a) How would you study road maintenance?

# Road maintenance (sketch)

1. Usage types (weight/axel) → road “quality” (capacities and free flow speed, intrinsic quality of the road). e.g. Highway Development and Management Model (**HDM-4**) of the World bank.
2. Data: Road capacities & free flow speed + O-D matrices + users' preferences → Usage (trip/no trip, mode, routes & time of usage)
3. Maintenance policy: What are the fixed and the variable costs? Which road should be repaired : when, where; intensity of repair? [Include possibly time for repairs, and impact of traffic during and after the repairs]
4. Output: road usage, congestion, user costs (travel time quality of the trip, SDC), attractivity and access to employment (and  $\Delta$ GD).  
→CBA and welfare analysis

# Case study 3: consumer behavior and waste

Discussion in class

# Modelling (open question)

(a) Formulate a rational shopping model which explains why households through away large quantities of food (and more) they buy